SYDNEY BOYS HIGH SCHOOL

MOORE PARK, SURRY HILLS

2015 HSC Assessment Task 1

Mathematics Extension 1

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen.
- Board approved calculators maybe used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Leave your answers in the simplest exact form, unless otherwise stated.
- In Questions 6-9, show relevant mathematical reasoning and/or calculations.
- A reference sheet has been provided.

Total Marks – 65

Section I (5 Marks)

Answer questions 1-5 on the Multiple Choice answer sheet provided.

Section II (60 Marks)

For Questions 6-9, start a new answer booklet for each question.

Examiner: *J. Chan*

Section I

5 marks

Attempt Questions 1-5

Use the multiple-choice answer sheet for Questions 1-5.

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1 What ratio does the point P (10, 11) divide the interval AB, where A (–2, 3) and B (7, 9)?

(A) 1 : 4

(B) 4 : –1

(C) 1 : –4

(D) 4 : 1

2 Two straight lines have gradients $m - 1$ and $m + 1$. The acute angle between the lines is θ. Which of the following is a correct expression for tan θ ?

(A) $\left|\dfrac{2}{m}\right|$

(B) $\left|\dfrac{2}{m^2}\right|$

(C) $\left|\dfrac{2}{2-m^2}\right|$

(D) $\left|\dfrac{2m}{2-m^2}\right|$

3 The polynomial $P(x) = x^4 - 3x^3 + ax^2 - 7x - 6$ leaves a remainder of 8 when divided by $(x + 1)$. What is the value of a?

(A) 3

(B) –3

(C) 4

(D) –4

4 If $\cos x = \frac{3}{4}$ and sin $x < 0$ then which of the following is the exact value of sin $2x$?

(A) $\frac{-3\sqrt{7}}{8}$

(B) $\frac{\sqrt{7}}{4}$

(C) $\frac{-\sqrt{7}}{4}$

(D) $\frac{3\sqrt{7}}{8}$

5 Given the roots of $x^2 - 2x - 1 = 0$ are $\tan \alpha$ and $\tan \beta$ where α and β are acute.
What is the value of $\alpha + \beta$?

(A) $\frac{-\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{-\pi}{4}$

End of Section I

Section II

60 marks

Attempt Questions 6-9

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 6-9, your responses should include relevant mathematical reasoning and/ or calculations.

Question 6 (15 marks) Use a SEPARATE writing booklet.

(a) Find the acute angle, to the nearest minute, between the lines **2**
$y = \frac{x}{4} - 2$ and $3x - y + 4 = 0$

(b) The curves $xy = 4$ and $x^2 = 16y$ intersect at (4, 1). **2**
Find the acute angle (to the nearest degree) between these two curves at (4, 1).

(c) Prove that $\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$ **2**

(d) Find the coordinates of the point P which divides AB externally in the ratio 2 : 1, **2**
where A is (3, –5) and B is (–4, 2).

(e) Sketch the region showing where the following inequalities hold simultaneously: **2**

$$y + 1 \le 0; \quad x + y - 1 < 0; \quad x \ge 2$$

(f) Determine the values of k and m such that the polynomial **3**
$P(x) = 2x^3 + kx^2 + mx - 3$ has factors $(x - 1)$ and $(x + 3)$.

(g) Find the equation of the third degree polynomial whose graph cuts **2**
the x-axis at –1, touches the x-axis at 1 and passes through the point (0, –2).

Question 7 (15 marks) Use a SEPARATE writing booklet.

(a) Simplify $\sin(270° - \theta) + \cos(-\theta)$ **2**

(b) If α, β, and γ are the roots of the equation $2x^3 - 5x^2 - 3x + 1 = 0$ find the values of:

(i) $(\alpha + \beta + \gamma) + \alpha\beta\gamma$. **1**

(ii) $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$. **1**

(iii) $\alpha^2 + \beta^2 + \gamma^2$. **2**

(iv) $\dfrac{\beta + \gamma}{\alpha} + \dfrac{\gamma + \alpha}{\beta} + \dfrac{\alpha + \beta}{\gamma}$. **2**

(c) (i) Let $t = \tan\dfrac{\theta}{2}$. Using the double-angle formulae, prove that: **2**

$$\sin\theta = \frac{2t}{1+t^2} \text{ and } \cos\theta = \frac{1-t^2}{1+t^2}$$

(ii) Use the substitution $t = \tan\dfrac{\theta}{2}$ to solve the equation (to the nearest degree): **3**

$$\sin\theta + 2\cos\theta + 2 = 0 \text{ for } 0° \le \theta \le 360°$$

(d) When the polynomial $P(x)$ is divided by $(x-1)(x-2)$ the remainder is $2x+3$. **2**
Find the remainder when $P(x)$ is divided by $(x-2)$.

Question 8 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Solve the inequality $\dfrac{x-5}{x-1} \le -1$ and graph the solution on a number line. **2**

(ii) Hence solve $\dfrac{\cos x - 4}{\cos x} \le -1$ for $0 \le x \le \pi$. **2**

(b) (i) Express $\sin x - \sqrt{3}\cos x$ in the form $A\sin(x+\beta)$, where $A > 0$ and $0 < \beta < 2\pi$. **2**

(ii) Hence solve $\sin x - \sqrt{3}\cos x = -1$ for $0 \le x < 2\pi$. **2**

(c) The equation $x^3 - mx + 2 = 0$ has two equal roots.

(i) Write the expressions for the sum of the roots and the product of roots. **1**

(ii) Hence find the value of m. **2**

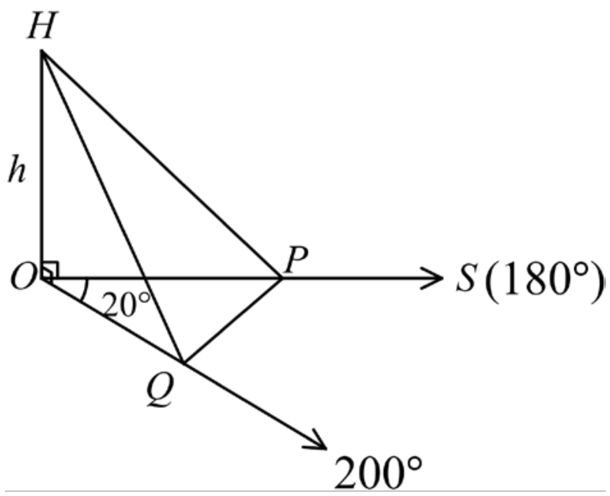
(d) Suppose $7\sin\theta - 24\cos\theta = R\sin(\theta - \alpha)$ where $R = 25$ and $\alpha = 73.7°$

(i) Let $x = 14\sin\theta - 48\cos\theta + 14$, use the above information to find the maximum and minimum values of x. **2**

(ii) Find the general solution for θ at which x attains its maximum. **2**

Question 9 (15 marks) Use a SEPARATE writing booklet.

(a) From a point P due south of a hill, the angle of elevation to the top of the hill is 33°. From another point Q bearing 200° from the hill, the angle of elevation is 44°. The distance PQ is 210 m.



(i) Express OP and OQ in terms of h. **1**

(ii) Calculate the height, h, of the hill to 3 significant figures. **2**

(b) The cubic equation $y = -2x^3 + px^2 + qx + 10$ has a stationary point at (–2, –10).

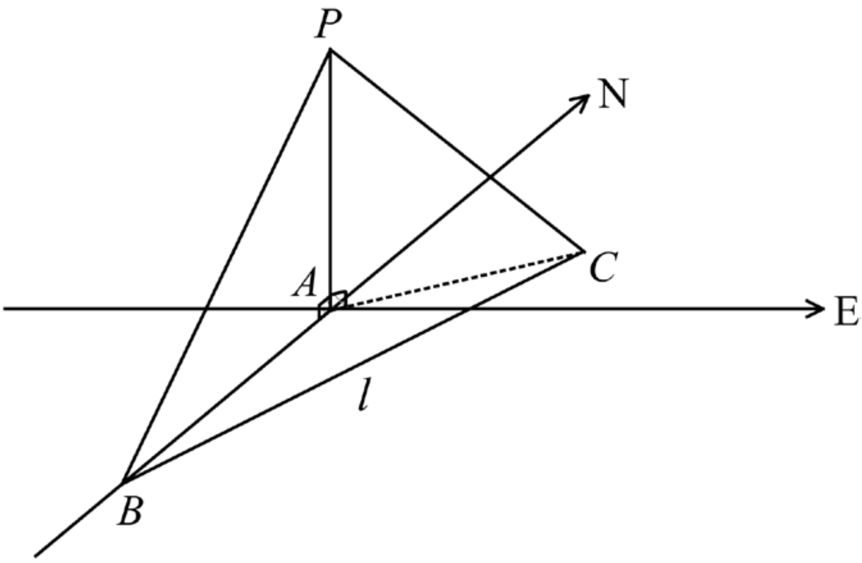
(i) Find the values of p and q and hence determine the position and nature of all stationary points (A sketch is not required). **3**

(ii) Show that if a cubic of the form $y = -2x^3 + px^2 + qx + 10$ is to have any stationary points at all, then it is necessary that $p^2 + 6q \geq 0$ **2**

Question 9 continues on page 7

Question 9 (continued)

(c) PA is a line perpendicular to the horizontal plane which contains the points A, B and C. **3**



The bearings of C from A and B are $\text{N}\,\theta\,\text{E}$ and $\text{N}\,\phi\,\text{E}$ respectively.

The length of BC is l.

If the angle of elevation of P from C is β, prove that $PA = \dfrac{l\sin\phi\tan\beta}{\sin\theta}$.

(d) (i) Simplify $\cos(A+B)+\cos(A-B)$. **1**

(ii) α and β are two acute angles satisfying the equation $6\cos^2\theta - 5\cos\theta + 1 = 0$. **3**

Using part (i) and without solving the equation, show that

$$\cos\left(\frac{\alpha+\beta}{2}\right)+\cos\left(\frac{\alpha-\beta}{2}\right)=\sqrt{2}$$

End of paper

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2015

HSC Task #1

Mathematics Extension 1

Suggested Solutions & Markers' Comments

QUESTION	Marker
1 – 5	JWC
6	AF
7	BK
8	DH
9	PSP

Multiple Choice Answers

1. B **2.** B **3.** A **4.** A **5.** A

2015 Mathematics Extension 1 HSC Assessment Task 1

Multiple Choices

Q1. The answer is B.

$$\frac{mx_2+nx_1}{m+n}=10$$
$$\frac{m(7)+n(-2)}{m+n}=10$$

$$7m-2n=10m+10n$$
$$\frac{m}{n}=-\frac{4}{1}$$

$$\therefore m:n=-4:1 \ or\ 4:-1$$

Q2. The answer is B.

$$\begin{aligned}\tan\theta &= \left|\frac{(m-1)-(m+1)}{1+(m^2-1)}\right| \\ &= \left|\frac{-2}{m^2}\right|\end{aligned}$$

Q3. The answer is A

$$\begin{aligned}P(-1)=(-1)^4-3(-1)^3+a(-1)^2-7(-1)-6&=8\\ 1-(-3)+a+7-6&=8\\ a&=3\end{aligned}$$

Q4. The answer is A

$$\begin{aligned}\sin 2x &= 2\sin x\cos x\\ &=2\left(\frac{-\sqrt{7}}{4}\right)\left(\frac{3}{4}\right)\\ &=\frac{-3\sqrt{7}}{8}\end{aligned}$$

Q5. The answer is A.

$$\alpha+\beta=\tan\alpha+\tan\beta=2$$
$$\alpha\beta=\tan\alpha\tan\beta=-1$$

$$\begin{aligned}\tan(\alpha+\beta)&=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\\ &=\frac{2}{1--1}\end{aligned}$$

$$\begin{aligned}\alpha+\beta&=\tan^{-1}(1)\\ &=\frac{\pi}{4}\end{aligned}$$

Question 6

a) $y = \frac{x}{4} - 2$ $\qquad$ $3x - y + 4 = 0$

$y = 3x + 4$

$m_1 = \frac{1}{4}$ $\qquad$ $m_2 = 3$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\left(\frac{1}{4}\right) - (3)}{1 + \left(\frac{1}{4}\right)(3)}\right|$$

$$= \frac{11}{7}$$

$$\theta \approx 57°32'$$

b) $xy = 4$ $\qquad$ $x^2 = 16y$

$y = \frac{4}{x}$ $\qquad$ $y = \frac{x^2}{16}$

$y = 4x^{-1}$ $\qquad$ $y' = \frac{x}{8}$

$y' = -4x^{-2}$

$= -\frac{4}{x^2}$

at $(4, 1)$

$m_1 = -\frac{4}{(4)^2}$ $\qquad$ $m_2 = \frac{(4)}{8}$

$= -\frac{1}{4}$ $\qquad$ $= \frac{1}{2}$

$$\tan\theta = \left|\frac{\left(-\frac{1}{4}\right) - \left(\frac{1}{2}\right)}{1 + \left(-\frac{1}{4}\right)\left(\frac{1}{2}\right)}\right|$$

$$= \frac{6}{7}$$

$$\theta \approx 41°$$

c) $LHS = \dfrac{\sin x + \sin 2x}{1 + \cos x + \cos 2x}$

$= \dfrac{\sin x + 2\sin x\cos x}{\cancel{1} + \cos x + 2\cos^2 x \cancel{- 1}}$

$= \dfrac{\sin x\cancel{(1 + 2\cos x)}}{\cos x\cancel{(1 + 2\cos x)}}$

$= \tan x$

$= RHS$

$\therefore \dfrac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$

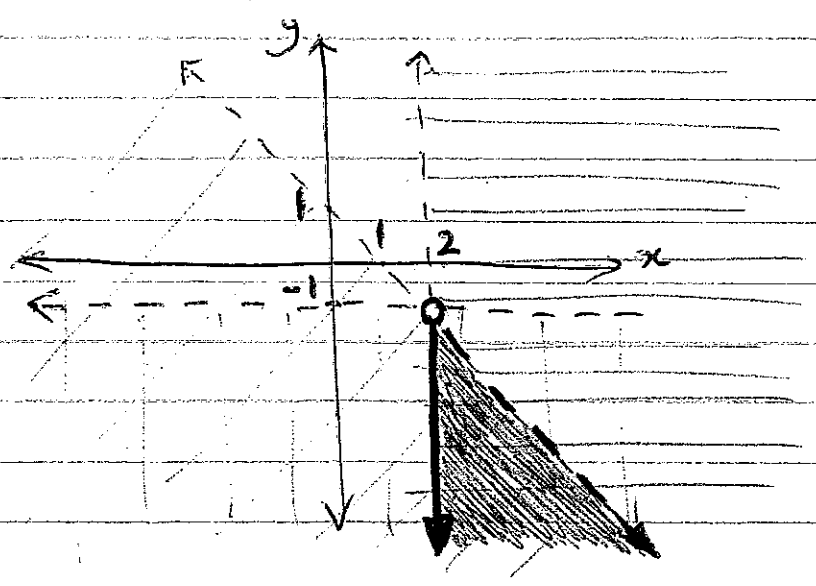
d) $A(3, -5)$ $B(-4, 2)$

$2 : -1$

$P\left(\dfrac{-1(3) + 2(-4)}{2 + (-1)}, \dfrac{-1(-5) + 2(2)}{2 + (-1)}\right)$

$P(-11, 9)$

e)



f) $P(x) = 2x^3 + kx^2 + mx - 3$

$$P(1) = 2(1)^3 + k(1)^2 + m(1) - 3 = 0$$
$$k + m - 1 = 0$$
$$k + m = 1 \quad \text{——— } ①$$

$$P(-3) = 2(-3)^3 + k(-3)^2 + m(-3) - 3 = 0$$

$$9k - 3m - 57 = 0$$
$$3k - m = 19 \quad \text{——— } ②$$

$$① + ②$$

$$4k = 20$$
$$\underline{k = 5}$$
sub into ①
$$(5) + m = 1$$
$$\underline{m = -4}$$

<u>OR</u>
since $(x-1)$ and $(x+3)$ are factors of $P(x)$
Let the roots of $2x^3 + kx^2 + mx - 3 = 0$ be
$1, -3$ & α.

$$(1)(-3)\alpha = -\frac{d}{a}$$
$$-3\alpha = \frac{-(-3)}{2}$$
$$-3\alpha = \frac{3}{2}$$
$$\alpha = -\frac{1}{2}$$

$$(1) + (-3) + \left(-\frac{1}{2}\right) = -\frac{b}{a}$$
$$-\frac{5}{2} = -\frac{k}{2}$$

$$\underline{k = 5}$$

$$(1)(-3) + (1)\left(-\tfrac{1}{2}\right) + (-3)\left(-\tfrac{1}{2}\right) = \frac{c}{a}$$

$$-2 = \frac{m}{2}$$

$$\underline{m = -4}$$

9) $P(x) = a(x+1)(x-1)^2$

$$P(0) = a\big((0)+1\big)\big((0)-1\big)^2 = -2$$

$$a = -2$$

$$\therefore P(x) = -2(x+1)(x-1)^2$$

OR

$$P(x) = -2x^3 + 2x^2 + 2x - 2$$

COMMENTS:

6)a)b) The acute angle between two lines is not given on the reference sheet.
It is important for students to memorise it or know that it is derived from

$$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$$

Note: $\tan(\theta + \phi)$ is given on the reference sheet

Many students had trouble with the formula. Many couldn't find the gradient of the tangents needed in (b).

c) The numerator should influence the decision students' make about $\cos 2x$. Many students stopped short of factorising.

d) Although the division of an interval in a given ratio is given on the reference sheet. Many mistakes were made.
m & n need to be paired with the correct coordinates.
One of them needs to be taken as negative to account for external division.

e) Note: In this situation, $y+1 \leq 0$ doesn't influence the intersection of $y+1\leq 0$; $x+y-1<0$; $x \geq 2$.
$y+1=0$, $x+y-1=0$ & $x=2$ are concurrent, intersecting at $(2,-1)$
This point should not be included in the region.

f) This was generally done well with most students gaining at least 2 marks.

g) Most students knew that the polynomial would involve $(x+1)(x-1)^2$.
However, instead of using $P(x) = a(x+1)(x-1)^2$ it appeared as though they considered $P(x) = (x+1)(x-1)^2 + k$

Question 7

(a) $\sin(270-\theta) + \cos(-\theta)$

$= -\cos\theta + \cos\theta$

$= 0$

Students mostly did not know their angles from the Yaxis.ie 90 or 270 plus or minus and angle.

(b) $2x^3 - 5x^2 - 3x + 1 = 0$

(i) Sum of roots $\alpha + \beta + \gamma = \frac{5}{2}$

Roots 2 at a time $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{-3}{2}$

Prod. of roots $\alpha\beta\gamma = -\frac{1}{2}$

Then $(\alpha + \beta + \gamma) + \alpha\beta\gamma$

$= \frac{5}{2} - \frac{1}{2}$

$= 2$

Done well.

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{-3/2}{-1/2} = 3$

Done well.

(iii) $\alpha^2 + \beta^2 + \gamma^2$

Consider $(\alpha+\beta+\gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + \alpha\beta + \alpha\gamma + \alpha\beta + \beta\gamma + \alpha\gamma + \beta\gamma$

$= \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= \left(\frac{5}{2}\right)^2 - 2\left(\frac{-3}{2}\right)$

$= \frac{25}{4} + 3 = \frac{37}{4}$

Generally done well but some students forgot to square the first term.

Q2 (b)(iv) $\frac{\beta+\gamma}{\alpha} + \frac{\gamma+\alpha}{\beta} + \frac{\alpha+\beta}{\gamma}$

$= \frac{\alpha+\beta+\gamma}{\alpha} + \frac{\alpha+\beta+\gamma}{\beta} + \frac{\alpha+\beta+\gamma}{\gamma} - 3$

$= (\alpha+\beta+\gamma)\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right) - 3$

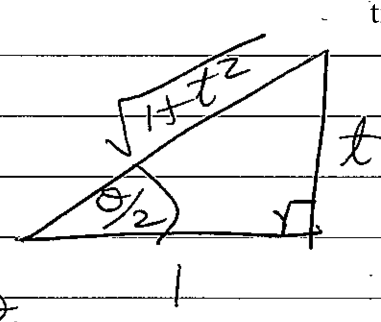
$= \frac{5}{2}(3) - 3$ from (ii)

$= \frac{15}{2} - 3$

$= \frac{9}{2}$

Only about a quarter of the students were able to manipulate the expression so it was in terms of the sums of roots(either 1 at a time,2 at a time of 3 at a time.)

(c) $t = \tan\frac{\theta}{2}$



Then $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$= 2 \times \frac{t}{\sqrt{1+t^2}} \times \frac{1}{\sqrt{1+t^2}}$

$= \frac{2t}{\sqrt{1+t^2}}$ ✓

Also $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

$= \frac{1}{1+t^2} - \frac{t^2}{1+t^2}$

$= \frac{1-t^2}{1+t^2}$ ✓

Mostly done well when attempted but about a quarter of students did not see how to proceed.

2.(c)(ii) $\sin\theta + 2\cos\theta + 2 = 0$ for $0^\circ \le \theta \le 360^\circ$

Let $t = \tan\frac{\theta}{2} \Rightarrow \frac{2t}{1+t^2} + \frac{2-2t^2}{1+t^2} + 2 = 0$

$$2t + 2 - \cancel{2t^2} + 2 + \cancel{2t^2} = 0.$$

$$2t + 4 = 0$$

$$t = -2$$

Common mistake was to not double the angle to find theta. Another common error was writing down 4 solutions for this equation - one for each quadrant. The most common error was not checking if 180 was a solution.

$$\Rightarrow \tan\frac{\theta}{2} = -2$$

$$\frac{\theta}{2} = 180 - 63^\circ 26' \text{ or } 360 - 63^\circ 26'$$

$$\frac{\theta}{2} = 116^\circ 34' \text{ or } 296^\circ 34'$$

$$\theta = 233^\circ 8'$$

too big.

Check $\theta = 180^\circ$

$$0 + {}^-2 + 2 = 0 \quad \checkmark$$

$$\therefore \theta = 180^\circ \text{ or } 233^\circ 8'$$

(d) $P(x) = (x-1)(x-2)Q(x) + 2x + 3.$

When divided by $(x-2)$, $R(x) = P(2)$

$$P(2) = 0 + 4 + 3$$

$$P(2) = 7$$

Generally done well but many did not know how to proceed.

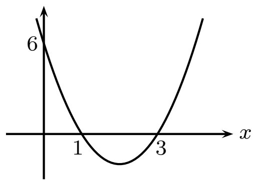
$\therefore$ remainder is 7.

2015 Year 11 Extension Task 1: Question 8 Solutions

8. (a) (i) Solve the inequality $\dfrac{x-5}{x-1} \leqslant -1$ and graph the solution on the number line. $\boxed{2}$

Solution: Method 1—

$$\begin{aligned}\left(\frac{x-5}{x-1}\right) \times (x-1)^2 &\leqslant -1 \times (x-1)^2,\ x \neq 1,\\ (x-1)(x-5) &\leqslant -(x-1)^2,\\ (x-1)\big([x-5]+[x-1]\big) &\leqslant 0,\\ (x-1)(2x-6) &\leqslant 0,\\ \therefore 1 < x &\leqslant 3.\end{aligned}$$



Method 2—

$$\begin{aligned}\text{Case 1: } x &< 1;\\ x-5 &\geqslant 1-x,\\ 2x &\geqslant 6,\\ x &\geqslant 3, \text{ a contradiction.}\\ \text{Case 2: } x &> 1;\\ x-5 &\leqslant 1-x,\\ 2x &\leqslant 6,\\ x &\leqslant 3,\\ \text{so } 1 < x &\leqslant 3, \text{ as above.}\end{aligned}$$

Comment: A number of candidates omitted the requested number line. Many failed to notice that $x \neq 1$.
Others succeeded in irritating the marker by putting an unnecessary equals (=) sign at the start of each line, probably under the mistaken impression that it means implies, the $\Longrightarrow$ sign, which is also unnecessary.

(ii) Hence solve $\dfrac{\cos x - 4}{\cos x} \leqslant -1$ for $0 \leqslant x \leqslant \pi$. $\boxed{2}$

Solution: Take old $x = \cos x + 1$, *i.e.* old $x - 1 = \cos x$,

$$\begin{aligned}\textit{i.e. } x-1 &= \cos x,\\ \therefore \cos x + 1 &> 1, \quad \text{and} \quad \cos x + 1 \leqslant 3,\\ \cos x &> 0, \qquad\qquad \cos x \leqslant 2.\\ \therefore 0 \leqslant x &< \frac{\pi}{2}.\end{aligned}$$

Comment: Hence means "*hence*", not "hence or otherwise": if the previous part is not used, full marks will not be given.
Some candidates are persisting in using columns which risks losing HSC marks if the marker loses track of the logic.
As a matter of courtesy, questions posed in terms of radians, like this one and the following two parts, should be answered in radians, and questions posed in terms of degrees, like part (d), should be answered in degrees.

(b) (i) Express $\sin x - \sqrt{3}\cos x$ in the form $A\sin(x+\beta)$ where $A > 0$ and $0 < \beta < 2\pi$. $\boxed{2}$

Solution:

$$\begin{aligned}
r &= \sqrt{1+3},\\
&= 2.\\
\therefore A\sin(x+\beta) &\equiv 2(\sin x\cos\beta + \cos x\sin\beta),\\
&= 2\left(\sin x.\frac{1}{2} + \cos x.\frac{-\sqrt{3}}{2}\right),\\
\sin\beta < 0,\ \cos\beta &> 0 \implies \beta \text{ in quadrant } 4,\\
\tan\beta &= -\sqrt{3},\\
\beta &= \frac{5\pi}{3}.\\
\text{So } \sin x - \sqrt{3}\cos x &= 2\sin\left(x + \frac{5\pi}{3}\right).
\end{aligned}$$

Comment: The need to have β positive caused a great deal of confusion, with much arbitrary fudging of results to get what seemed to look right.
Putting down several guesses, in the hope that one of them might hit the jackpot, is not a successful method.
Legibility, particularly of π, has been a frequent problem: $\pi \neq \Omega$ or $\cap$.
If the marker cannot read what is scrawled, no mark can be given.

(ii) Hence solve $\sin x - \sqrt{3}\cos x = -1$ for $0 \leqslant x < 2\pi$. $\boxed{2}$

Solution:

$$\begin{aligned}
2\sin\left(x + \frac{5\pi}{3}\right) &= -1,\\
\sin\left(x + \frac{5\pi}{3}\right) &= -\frac{1}{2},\\
&= \sin\left(-\frac{\pi}{6}\right),\\
x + \frac{5\pi}{3} &= \cdots \cancel{-\frac{\pi}{6}}, \cancel{\frac{7\pi}{6}}, \frac{11\pi}{6}, \frac{19\pi}{6}, \cancel{\frac{23\pi}{6}}, \ldots\\
x &= \frac{\pi}{6}, \frac{3\pi}{2}.
\end{aligned}$$

Comment: This part, which demanded a restricted set of solutions, gave more problems than the later $\big($(d)(ii)$\big)$ general solutions.
Many "correct working after error" marks were, nevertheless, gained.

(c) The equation $x^3 - mx + 2 = 0$ has two equal roots.

(i) Write the expressions for the sum of the roots and the product of the roots. $\boxed{1}$

Solution: Let roots be α, α, β;

$$\begin{aligned}
2\alpha + \beta &= 0 \ \ldots\ldots\ldots\ldots \boxed{1}\\
\alpha^2\beta &= -2 \ldots\ldots\ldots\ldots \boxed{2}
\end{aligned}$$

Comment: This was generally well done. Some candidates thought that this cubic had *only* two equal roots which, unfortunately, meant they had little chance of gaining any marks in the following part.

(ii) Hence find the value of m. $\boxed{2}$

Solution:

$$\begin{aligned} \text{From } \boxed{1}: \quad \beta &= -2\alpha, \\ \text{sub. in } \boxed{2}: \quad -2\alpha^3 &= -2, \\ \text{so } \alpha &= 1, \\ \beta &= -2. \\ -m &= \alpha^2 + 2\alpha\beta, \\ &= 1 - 4, \\ \therefore m &= 3. \end{aligned}$$

Comment: Some candidates again failed to realise that HENCE means that the earlier result must be used.

(d) Suppose $7\sin\theta - 24\cos\theta = R\sin(\theta - \alpha)$ where $R = 25$ and $\alpha = 73.7°$.

(i) Let $x = 14\sin\theta - 48\cos\theta + 14$, use the above information to find the maximum and minimum values of x. $\boxed{2}$

Solution:

$$\begin{aligned} x &= 2(7\sin\theta - 24\cos\theta) + 14, \\ &= 50\sin(\theta - 73.7°) + 14. \\ \therefore \text{Maximum } x &= 50 + 14, \\ &= 64, \\ \text{minimum } x &= -50 + 14, \\ &= -36. \end{aligned}$$

Comment: This was well answered by those who used the year 9 or 10 method of seeing that sine varies from 1 to -1.
Differentiating once or twice only added more chances to make an error—after finding the values of θ, candidates often forgot to find x—
i.e. to answer the question.

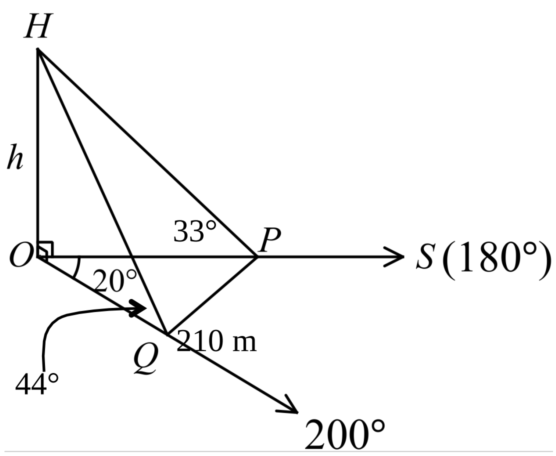
(ii) Find the general solution for θ at which x attains its maximum. $\boxed{2}$

Solution:

$$\begin{aligned} \sin(x - 73.7°) &= 1 \text{ for a maximum}, \\ \theta - 73.7° &= 180°n + (-1)^n(90°), \\ i.e.\ \theta &= 180°n + (-1)^n(90°) + 73.7°, \\ &= 163.7° + 360°n. \end{aligned}$$

Comment: This part was well answered by most candidates who got this far.
It would be appreciated by the marker if answers could be entirely given in the same form as the question, *i.e.* do not mix radians and degrees—$163.7° + 2\pi n$ is slightly sick-making.

(a) From a point P due south of a hill, the angle of elevation to the top of the hill is 33°.
From another point Q bearing 200° from the hill, the angle of elevation is 44°.
The distance PQ is 210 m.



(i) Express OP and OQ in terms of h. **1**

In Δ OHP: $\tan 33° = \dfrac{OH}{OP}$

$\therefore OP = \dfrac{OH}{\tan 33°} = h\cot 33° = h\tan 57°$

Similarly, $OQ = h \tan 46°$

Comment:
Many students made the calculator work in the next part harder by not considering the form in which they left OP and OQ.

(ii) Calculate the height, h, of the hill to 3 significant figures. **2**

Applying the cosine rule to Δ OQP:
$QP^2 = OQ^2 + OP^2 - 2OP.OQ\cos 20°$

$$210^2 = h^2\tan^2 46° + h^2\tan^2 57° - 2(h\tan 46°)(h\tan 57°)\cos 20°$$
$$= h^2(\tan^2 46° + \tan^2 57° - 2\tan 46°\tan 57°\cos 20°)$$

$$\therefore h^2 = \frac{210^2}{\tan^2 46° + \tan^2 57° - 2\tan 46°\tan 57°\cos 20°}$$
$$\therefore h = \frac{210}{\sqrt{\tan^2 46° + \tan^2 57° - 2\tan 46°\tan 57°\cos 20°}}$$
$$\doteqdot 314.2097$$
$$\doteqdot 314\text{ m }(3\text{ sf})$$

Comment:
Students who started with $\cos\theta = \dfrac{a^2+b^2-c^2}{2ab}$ approach, generally ended up making more algebraic mistakes. Too many students cannot do the calculator part of this question well.
Students have to check that they have answered the question answered. Fortunately, this time, there was no penalty for disregarding the significant figures component of the question.

(b) The cubic equation $y = -2x^3 + px^2 + qx + 10$ has a stationary point at (–2, –10).

(i) Find the values of p and q and hence determine the position and nature of all stationary points (A sketch is not required). **3**

The point (–2, –10) lies on the cubic i.e. $-10 = 16 + 4p - 2q + 10$

$\boxed{\therefore 2p - q = -18 \qquad -(1)}$ ✓ $\frac{1}{2}$

$\frac{dy}{dx} = -6x^2 + 2px + q$

(–2, 0) lies on the derivative of the cubic, $\frac{dy}{dx}$ i.e. $0 = -24 - 4p + q$

$\boxed{\therefore 4p - q = -24 \qquad -(2)}$ ✓ $\frac{1}{2}$

(1) – (2): $-2p = 6$

$\therefore\ p = -3$

Substituting into (1): $-6 - q = -18$

$\therefore\ q = 12$

$\boxed{\therefore\ p = -3,\ q = 12}$

✓ 1
No $\frac{1}{2}$ marks

Stationary points occur when $\frac{dy}{dx} = -6x^2 - 6x + 12 = 0$

As x = –2 is a root of this equation then as the sum of both roots is –1, the other root is x = 1 i.e. (1, 17)

As $\frac{d^2y}{dx^2} = -12x - 6x$, then (–2,-10) is a minimum turning point and (1, 17) is a maximum turning point. ✓ 1

Comment:
Many students did not see the second part of this question.
Many students made many unexpected algebraic errors in this question.
Many students preferred the first derivative method for testing maxima/minima rather than the time efficient second derivative method.

(ii) Show that if a cubic of the form $y = -2x^3 + px^2 + qx + 10$ is to have any stationary points at all, then it is necessary that $p^2 + 6q \geq 0$ **2**

$\frac{dy}{dx} = -6x^2 + 2px + q$

If there are going to be stationary points then $\frac{dy}{dx} = -6x^2 + 2px + q = 0$ and $\Delta \geq 0$

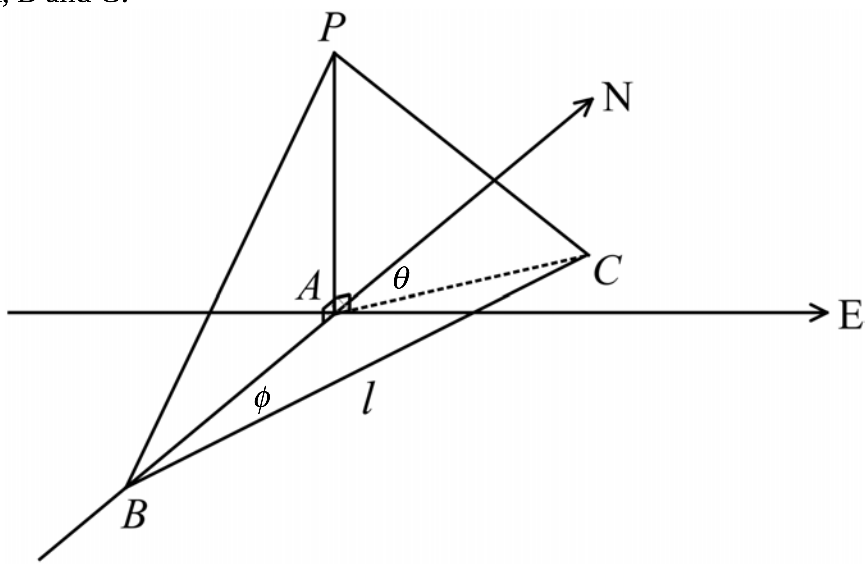
$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= 4p^2 - 4(-6)(q) \\ &= 4(p^2 + 6q)\end{aligned}$$

$\Delta \geq 0 \Rightarrow 4(p^2 + 6q) \geq 0$

$\therefore p^2 + 6q \geq 0$

Comment:
Generally well done. Students who didn't explain where the inequality came from were penalised. To explain students had to indicate that $\Delta \geq 0$ or $b^2 - 4ac \geq 0$ or equivalent.

(c) PA is a line perpendicular to the horizontal plane which contains the points A, B and C. **3**



The bearings of C from A and B are $\text{N}\,\theta\,\text{E}$ and $\text{N}\,\phi\,\text{E}$ respectively.
The length of BC is l.

If the angle of elevation of P from C is β, prove that $PA = \dfrac{l\sin\phi\tan\beta}{\sin\theta}$.

In $\Delta\, PAC$:

$$\tan\beta = \frac{PA}{CA}$$

$$\therefore CA = \frac{PA}{\tan\beta}$$

Now $\angle BAC = \pi - \theta$ (straight line)

Applying the sine rule to $\Delta\, BAC$:

$$\frac{CA}{\sin\phi} = \frac{l}{\sin(\pi - \theta)}$$

$$\therefore \frac{\frac{PA}{\tan\beta}}{\sin\phi} = \frac{l}{\sin\theta} \Rightarrow \frac{PA}{\sin\phi\tan\beta} = \frac{l}{\sin\theta}$$

$$\therefore PA = \frac{l\sin\phi\tan\beta}{\sin\theta}$$

Comment:
Generally well done.
Students that didn't make reference to $\angle BAC = \pi - \theta$ or $\angle BAC = 180° - \theta$ lost 1 mark.

(d) (i) Simplify $\cos(A+B)+\cos(A-B)$. **1**

$$\cos(A+B)+\cos(A-B)=\cos A\cos B-\sin A\sin B+\cos A\cos B+\sin A\sin B$$
$$=2\cos A\cos B$$

Comment:

Generally well done.

(ii) α and β are two acute angles satisfying the equation $6\cos^2\theta-5\cos\theta+1=0$. **3**

Using part (i) and without solving the equation, show that

$$\cos\left(\frac{\alpha+\beta}{2}\right)+\cos\left(\frac{\alpha-\beta}{2}\right)=\sqrt{2}$$

$$6\cos^2\theta-5\cos\theta+1=0\Rightarrow\begin{cases}\cos\alpha+\cos\beta=\frac{5}{6} & -(1)\\ \cos\alpha\cos\beta=\frac{1}{6} & -(2)\end{cases}$$

$\boxed{\checkmark\ \tfrac{1}{2}}$

From (i) we get the following:

$$\cos\left(\frac{\alpha+\beta}{2}\right)+\cos\left(\frac{\alpha-\beta}{2}\right)=\cos\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)+\cos\left(\frac{\alpha}{2}-\frac{\beta}{2}\right)$$
$$=2\cos\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right)\qquad -(3)$$

$\boxed{\checkmark\ \tfrac{1}{2}}$

From (1):

$$\cos\alpha+\cos\beta=\frac{5}{6}\Rightarrow 2\cos^2\frac{\alpha}{2}-1+2\cos^2\frac{\beta}{2}-1=\frac{5}{6}$$
$$\therefore 2\left(\cos^2\frac{\alpha}{2}+\cos^2\frac{\beta}{2}\right)=2\frac{5}{6}$$

From (2):

$$\cos\alpha\cos\beta=\frac{1}{6}\Rightarrow\left(2\cos^2\frac{\alpha}{2}-1\right)\left(2\cos^2\frac{\beta}{2}-1\right)=\frac{1}{6}$$
$$\therefore 4\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}-2\left(\cos^2\frac{\alpha}{2}+\cos^2\frac{\beta}{2}\right)+1=\frac{1}{6}$$
$$\therefore 4\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}=2\times 2\frac{5}{6}+\frac{1}{6}-1$$
$$\therefore 4\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}=2$$
$$\therefore 2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}=\sqrt{2}\qquad\left[0<\alpha,\beta<\frac{\pi}{2}\right]$$

So substituting into (3) we get

$$\cos\left(\frac{\alpha+\beta}{2}\right)+\cos\left(\frac{\alpha-\beta}{2}\right)=2\cos\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right)$$
$$=\sqrt{2}$$

Alternative Solution

$$6\cos^2\theta - 5\cos\theta + 1 = 0 \Rightarrow \begin{cases} \cos\alpha + \cos\beta = \dfrac{5}{6} & -(1) \\ \cos\alpha\cos\beta = \dfrac{1}{6} & -(2) \end{cases}$$

$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1 \Rightarrow \cos\frac{\alpha}{2} = \sqrt{\dfrac{1+\cos\alpha}{2}}$ [positive square root as α is acute]

Similarly, $\cos\frac{\beta}{2} = \sqrt{\dfrac{1+\cos\beta}{2}}$.

$$\begin{aligned} \cos\left(\frac{\alpha+\beta}{2}\right) + \cos\left(\frac{\alpha-\beta}{2}\right) &= \cos\left(\frac{\alpha}{2}+\frac{\beta}{2}\right) + \cos\left(\frac{\alpha}{2}-\frac{\beta}{2}\right) \\ &= 2\cos\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right) && \text{[from (i)]} \\ &= 2\times\sqrt{\frac{1+\cos\alpha}{2}}\times\sqrt{\frac{1+\cos\beta}{2}} \\ &= \sqrt{(1+\cos\alpha)(1+\cos\beta)} \\ &= \sqrt{1+\cos\alpha+\cos\beta+\cos\alpha\cos\beta} && \text{[from (1) and (2)]} \\ &= \sqrt{1+\tfrac{5}{6}+\tfrac{1}{6}} \\ &= \sqrt{2} \end{aligned}$$

Comment:

Many students didn't realise that if α and β are two acute angles satisfying the equation then:

$$\cos\alpha + \cos\beta = \tfrac{5}{6} \text{ and } \cos\alpha\cos\beta = \tfrac{1}{6}.$$

Writing $\alpha + \beta = \frac{5}{6}$ and $\alpha\beta = \frac{1}{6}$ was a significant error that meant a student could not earn any marks from work that derived from this.

Not many students were able to get to $\cos\left(\dfrac{\alpha+\beta}{2}\right) + \cos\left(\dfrac{\alpha-\beta}{2}\right) = 2\cos\left(\dfrac{\alpha}{2}\right)\cos\left(\dfrac{\beta}{2}\right)$ or if they did they did not realise the implication about now using the double angle results for $\cos\alpha$.